Pre-Calculus Notes  
Section 9.1 - Sequences and Series DAY TWO

Now let's look at the sum of a sequence - called a SERIES.

Consider the following sequence.
(A) What is its explicit formula?
(B) Write the sequence as a series - sum of the 4 terms.
(C) Find the actual sum of the series.

Ex. 1. \(3, 6, 9, 12\)

(A) \(a_n = 3n\)

(B) Series: \(3 + 6 + 9 + 12\)

(C) \(S_4\) (sum of 4 terms) = 30

Now let's look at the shorthand notation developed to indicate a series.

\[
\sum_{n=1}^{4} 3n = \sum \begin{array}{cccc}
1 & 2 & 3 & 4 \\
3 & 6 & 9 & 12
\end{array}
\]

\[S_4 = 30\]

We call this form **SUMMATION NOTATION** or **SIGMA NOTATION**

Here, \(n\) is the index of summation (the variable here is \(n\), but can be any letter of the alphabet)

1 is the lower limit of summation (this is usually one but can be any whole number)

4 is the upper limit of summation (this is usually a whole number, but can be \(\infty\))

Find the following series in **EXPANDED FORM** first and then find the actual sum of the series

Ex. 2. \(\sum_{k=1}^{5} 3n-1 = \sum \begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
3 & 5 & 8 & 11 & 14
\end{array}\)

\[S_5 = 40\]

Ex. 3. \(\sum_{i=3}^{7} 1+i^2 = \sum \begin{array}{cccc}
3 & 4 & 5 & 6 \\
10 & 17 & 26 & 37
\end{array}\)

\[S_5 = 140\]

Now, let’s look at how we can use the calculator to get the sum of a series.

**Calculator:** sum (seq \(\{1+x^2, x, 3, 7, 1\}\)) = \(\boxed{140}\)

**Steps:**
1. \(2^{nd} \text{ STAT} \) ListMath  \(\rightarrow\) List,OPS  \(\rightarrow\) \(y_1 = 1 + x^2\)
2. \(2^{nd} \text{ STAT} \) List,OPS  \(\rightarrow\) \(\text{seq} \) \(\{y_1, x, 3, 7, 1\}\)  \(\rightarrow\) \(\text{Vars, Y-Vars}\)  \(\rightarrow\) \(140\)

**If you have put \(1 + x^2\) into \(y_1\):**

**Equation:** \(1 + x^2\)

**Limits of summation:**

**Var. X**  \(\rightarrow\) go by ones

**Limits of summation:**

If you have put \(1 + x^2\) into \(y_1\):
Now, let's reverse the process and go from expanded form back to Sigma Notation.

Ex. 4. \[ \frac{1}{3(1)+5} + \frac{1}{3(2)+5} + \frac{1}{3(3)+5} + \cdots + \frac{1}{3(9)+5} = \sum_{i=1}^{9} \frac{1}{3i+5} \]

Ex. 5. \[ 3\left( \frac{1}{5} \right) + 1 + 3\left( \frac{2}{5} \right) + 2 + \cdots + 3\left( \frac{8}{5} \right) + 8 = \sum_{i=1}^{8} 3\left( \frac{i}{5} + 1 \right) \]

Ex. 6. \[ 4 + (-16) + 64 + (-256) + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} (4)^n \]

Ex. 7. \[ 10 + 10 + 10 + 10 + 10 = \sum_{k=1}^{5} 10 \]

NOW - Do you remember... ! - factorial!

Ex. 8. \[ 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \]

Simplify the following expressions.

Ex. 9. \[ \frac{5!}{7!} = \frac{1}{42} \]

Ex. 10. \[ \frac{98!}{100!} = \frac{98 \cdot 97 \cdot 96 \cdot \cdots}{100} = \frac{1}{9900} \]

Ex. 11. \[ \frac{(n)}{(n+1)!} = \frac{\frac{\text{Product of the first } n \text{ natural numbers}}{(n+1)!}}{1} \]

Ex. 12. \[ \frac{(n-2)}{n!} = \frac{\frac{(n-2)!}{n!}}{1} = \frac{1}{n(n-1)} \]

Using this knowledge, write the first 5 terms of the following sequence:

Ex. 13. \[ a_n = \frac{2^n}{n!} \]

\[ \frac{2}{1}, \frac{4}{2}, \frac{8}{6}, \frac{16}{24}, \frac{32}{120} \]

Could you use “table” on your calculator here? 

For the following series
(a) write it in sigma notation
(b) find the indicated PARTIAL SUM - sum of part of the series

\[ 1 + \frac{2}{2} + \frac{2^2}{4} + \frac{2^3}{8} + \frac{2^4}{16} + \frac{2^5}{32} + \cdots = \sum_{i=1}^{\infty} \frac{1}{2^i} \]

Ex. 14. \[ \frac{1}{2} + \frac{2}{4} + \frac{6}{8} + \frac{24}{16} + \frac{120}{32} + \cdots = \sum_{i=1}^{\infty} \frac{1}{2^i} \]

AND \[ S_3 = \frac{7}{4} \]

(b)