DAY ONE Examples: Solve the following algebraically (no calculator). Then verify your answers with a calculator. **SHOW YOUR WORK.**

<table>
<thead>
<tr>
<th>Algebraically Solved</th>
<th>Solution(s) Verified with Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \sin x = \frac{\sqrt{3}}{2} )</td>
<td>1. We could use a couple of techniques for checking our solution.</td>
</tr>
<tr>
<td>( x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}, n \in \mathbb{Z} )</td>
<td>Window:</td>
</tr>
<tr>
<td>( x_{\text{min}} = -\frac{3\pi}{2}, x_{\text{max}} = \frac{3\pi}{2}, x_{\text{scf}} = \frac{\pi}{2} )</td>
<td>( y_{\text{min}} = -1, y_{\text{max}} = 1, y_{\text{scf}} = 0 )</td>
</tr>
<tr>
<td><strong>a)</strong> Intersect: ( y_1 = \sin x ) ( y_2 = \frac{\sqrt{3}}{2} )</td>
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<tr>
<td><strong>b)</strong> Zero: ( y_1 = \sin x - \frac{\sqrt{3}}{2} ) (You MUST set the equation to 0)</td>
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<tr>
<td>( x = \frac{\pi}{6} \text{ or } \frac{\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6} )</td>
<td>2. Now check your answer by graphing the function and using one of the techniques seen in Example 1.</td>
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<td>( \sqrt{3} \approx 3.927 )</td>
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<td>( \frac{5\pi}{6} \approx 5.498 )</td>
<td>2. Now check your answer by graphing the function and using one of the techniques seen in Example 1.</td>
</tr>
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</table>

| 2. \( \sin x + \sqrt{2} = -\sin x \) | HINT: Collect like terms. |
| \( \sin x + \sin x = -\sqrt{2} \) | \( x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4} \) |
| \( 2 \sin x = -\sqrt{2} \) | \( x = \frac{\pi}{4} \text{ or } \frac{5\pi}{4} \) |
| \( \sin x = -\frac{\sqrt{2}}{2} \) | \( x = \frac{\pi}{4} \text{ or } \frac{5\pi}{4} \) |

| 3. \( \tan x \cos^2 x = 2 \tan x \) | HINT: Set equal to zero and factor out a GCF. |
| \( \tan x \cos^2 x - 2 \tan x = 0 \) | \( x = 0, \pi n, n \in \mathbb{Z} \) |
| \( \tan x (\cos^2 x - 2) = 0 \) | 3. Again, it is good practice to verify your solutions! 😊 |
| \( \tan x = 0 \) or \( \sqrt{\cos^2 x} = \sqrt{2} \) | \( 0 = 0 \) \( 0 = 0 \) |
| \( x = 0 \) or \( \cos x = \pm \sqrt{2} \) | \( \pi \approx 3.14 \) \( \pi \approx 3.14 \) |

- **better way to write:** \( x = \pi n, n \in \mathbb{Z} \)
4. \(3 \tan^2 x - 1 = 0\) over the interval \((0, 2\pi)\)

**HINT:** Solve for \(\tan^2 x\) and take the square root of both sides.

\[
3 \tan^2 x = 1 \\
\tan x = \pm \frac{\sqrt{3}}{3}
\]

\(Q, I, II, III, IV\)

\[
\theta = \frac{\pi}{6} \quad \text{or} \quad x = \frac{7\pi}{6}
\]

4. Now **ALGEBRAICALLY** verify that your answers are correct.

\[
\frac{\pi}{6} \approx 0.52 \\
\frac{5\pi}{6} \approx 2.62 \\
\frac{7\pi}{6} \approx 3.67 \\
\frac{11\pi}{6} \approx 5.76
\]

5. \(2 \sin^2 x - \sin x - 1 = 0\) over the interval \((0, 2\pi)\)

**HINT:** Factor as a quadratic or use the quadratic formula.

\[
(2 \sin x + 1)(\sin x - 1) = 0
\]

\[
2 \sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0
\]

\[
2 \sin x = -1 \quad \text{or} \quad \sin x = 1
\]

\[
\sin x = -\frac{1}{2} \quad \text{or} \quad x = \frac{\pi}{2}
\]

\(Q, IV, III\)

\[
\theta = \frac{3\pi}{6} \quad \text{or} \quad \frac{11\pi}{6}
\]

5. Again, check your answer! Let’s not make silly mistakes. 😊

\[
\frac{\pi}{2} \approx 1.57 \\
\frac{3\pi}{2} \approx 3.07 \\
\frac{5\pi}{6} \approx 5.76
\]

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**DAY TWO Examples:** Solve the following equations involving **MULTIPLE angles**

1. \(2 \cos(3t) - 1 = 0\)

\[
2 \cos(3t) = 1 \\
\cos(3t) = \frac{1}{2}
\]

\(Q, I, IV\)

\[
3t = \frac{\pi}{3} + \frac{2\pi}{3}n \quad \text{or} \quad 3t = \frac{5\pi}{3} + \frac{2\pi}{3}n
\]

\[
t = \frac{\pi}{9} + \frac{2\pi}{3}n \quad \text{or} \quad \frac{5\pi}{9} + \frac{2\pi}{3}n
\]

\(n \in \mathbb{Z}\)

---

2. \(3 \tan \left(\frac{x}{2}\right) + 3 = 0\)

\[
3 \tan \left(\frac{x}{2}\right) = -3 \\
\tan \left(\frac{x}{2}\right) = -1
\]

\[
\frac{x}{2} = \frac{3\pi}{4} + n\pi
\]

\[
x = \frac{3\pi}{2} + 2n\pi
\]
3. \(2\sin^2 x + 3\cos x - 3 = 0\)
   **HINT:** Solve by rewriting & eliminating a function.
   
   \[
   2(1 - \cos^2 x) + 3\cos x - 3 = 0 \\
   2 - 2\cos^2 x + 3\cos x - 3 = 0 \\
   2\cos^2 x - 3\cos x + 1 = 0 \\
   (2\cos x - 1)(\cos x - 1) = 0 \\
   2\cos x - 1 = 0 \\
   \cos x = \frac{1}{2} \\
   x = \frac{\pi}{3}, \frac{5\pi}{3} \\
   
   \]

4. \(\sec^2 x - 2\tan x = 4\)
   **HINT:** Solve by using the "inverse" function.
   
   \[
   (1 + \tan^2 x) - 2\tan x = 4 \\
   \tan^2 x - 2\tan x - 3 = 0 \\
   (\tan x - 3)(\tan x + 1) = 0 \\
   \tan x = 3 \quad \text{or} \quad \tan x = -1 \\
   \]

   **QI, IV** Period tangent = \(\pi\)

   \[
   \tan^{-1} \frac{3}{2} \quad \text{or} \quad \tan^{-1} \frac{1}{3} \\
   x = \tan^{-1} 3 + n\pi \\
   \text{and} \quad x = -\frac{\pi}{3} + n\pi \\
   \]

5. \(4\sin^3 x - 2\sin^2 x - 2\sin x = 0\)
   Solve by using a calculator. Approximate solutions to 3 decimal places over the interval \([0, 2\pi]\).
   
   \[
   x \approx 4.120 \quad \text{or} \quad x \approx 5.305 \\
   \]

6. \(\csc^2 x + 0.5\cot x - 5 = 0\)
   Solve by using a calculator. Approximate solutions to 3 decimal places over the interval \([0, 2\pi]\).
   
   \[
   1 + \cot^2 x + 0.5\cot x - 5 = 0 \\
   \cot^2 x + 0.5\cot x - 4 = 0 \\
   \]

   **c** 
   
   \[
   \cot x = -\frac{1.5 \pm \sqrt{5^2 - 4(-4)}}{2} \\
   \cot x = \frac{-1.5 \pm \sqrt{25 + 16}}{2} \\
   \cot x \approx 1.766 \quad \text{or} \quad \cot x \approx -2.266 \\
   \tan x \approx \frac{1}{1.766} \quad \tan x \approx \frac{1}{-2.266} \\
   \]

   \[
   x \approx 0.515 + n\pi \quad \text{or} \quad x \approx 2.723 + n\pi \\
   \]

**DAY THREE Examples:**

Solve the following by using the quadratic formula, then graph and check your solutions.

1. \(3\tan^2 x + 4\tan x - 4 = 0\)
   
   \[
   \tan x = \frac{-4 \pm \sqrt{16 - 4(-4)}}{6} \\
   \tan x = \frac{-4 \pm 8}{6} \\
   \tan x = \frac{2}{3} \quad \text{or} \quad \tan x = -2 \\
   \]

   \[
   x \approx \frac{2\pi}{3}, \pi \text{ or } x \approx 2.034 + n\pi \\
   \]

2. \(\csc^2 x + 0.5\cot x - 5 = 0\)
   
   \[
   1 + \cot^2 x + 0.5\cot x - 5 = 0 \\
   \cot^2 x + 0.5\cot x - 4 = 0 \\
   \]

   **c** 
   
   \[
   \cot x = -\frac{1.5 \pm \sqrt{5^2 - 4(-4)}}{2} \\
   \cot x = \frac{-1.5 \pm \sqrt{25 + 16}}{2} \\
   \cot x \approx 1.766 \quad \text{or} \quad \cot x \approx -2.266 \\
   \tan x \approx \frac{1}{1.766} \quad \tan x \approx \frac{1}{-2.266} \\
   \]

   \[
   x \approx 0.515 + n\pi \quad \text{or} \quad x \approx 2.723 + n\pi \\
   \]
3. The monthly sales $S$ (in hundreds of units) of skiing equipment are approximated by $S = 58.3 + 32.5 \cos \left( \frac{\pi}{6} \cdot t \right)$, where $t$ is the time (in months), with $t = 1$ corresponding to January. Determine the months when the sales exceed 7500 units. 

$$S = \frac{7500}{100} = 75$$

$$75 = 58.3 + 32.5 \cos \left( \frac{\pi}{6} \cdot t \right)$$

Calculate intersection points:

$$y_1 = 58.3 + 32.5 \cos \left( \frac{\pi}{6} \cdot t \right)$$

$$y_2 = 75$$

Intersection points: $(1.97, 75)$ and $(10.03, 75)$

Exceeding is above line $y = 75$

January, November, December

4. A sharpshooter intends to hit a target at a distance of 1000 yds. With a gun that has a muzzle velocity of 1200 ft. per second. Neglecting air resistance, determine the gun's minimum angle of elevation $\theta$ if the range is given by the function $r = \frac{1}{32} v_0^2 \sin 2\theta$.

$$1000 \text{ yards} = 3000 \text{ feet}$$

$$3000 = \frac{1}{32} (1200^2)(\sin 2\theta)$$

$$y_1 = \frac{1}{32} (1200^2)(\sin 2\theta)$$

$$y_2 = 3000$$

Intersection point: $(1.9, 3000)$

$1.9^\circ$