Since the exponential function $f(x) = b^x$ is one-to-one, it has an inverse function. The inverse function of an exponential function is called a logarithmic function.

**MEMORIZE:**

If $x = b^y$, then $y = \log_b x$. **AND** If $y = \log_b x$, then $x = b^y$.

**EXAMPLE 1:** Graph the function in **ONE COLOR**. Then graph its INVERSE in a **SECOND COLOR**.

| ORIGINAL FUNCTION: $y = 2^x$ |
| Domain: $(-\infty, \infty)$ | Range: $(0, \infty)$ |
| X-Intercepts: **None** |
| Y-Intercepts: $(0, 1)$ |
| Increasing or Decreasing? **Increasing** |
| Equation of Asymptote: $y = 0$ |

| INVERSE FUNCTION: $y = \log_2 x$ |
| Domain: $(0, \infty)$ | Range: $(-\infty, \infty)$ |
| X-Intercepts: $(1, 0)$ |
| Y-Intercepts: **None** |
| Increasing or Decreasing? **Increasing** |
| Equation of Asymptote: $x = 0$ |

**MEMORIZE:**

A logarithm is just an exponent! 😊

A logarithm with a base of 10 is a common logarithm. So, instead of writing $\log_{10} x$, we will write $\log x$.

A logarithm with a base of $e$ is a natural logarithm. So, instead of writing $\log_e x$, we will write $\ln x$.

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$ and $e \approx 2.718281828$...

Example 2: Rewrite each expression in logarithmic form.

a. $4^3 = 64$  
   \[\log_4 64 = 3\]

b. $10^3 = 1000$  
   \[\log_{10} 1000 = 3\]

c. $e^{-2} \approx 0.14$  
   \[\log_e 0.14 \approx -2\]
Example 3: Rewrite each expression in exponential form.

a. \( \ln 2 \approx 0.70 \) 
   \[ e^{0.70} = 2 \]

b. \( \log_5 125 = 3 \) 
   \[ 5^3 = 125 \]

c. \( \log_{10} 0.1 = -1 \) 
   \[ 10^{-1} = 0.1 \]

Example 4: Use the definition of logarithmic function to evaluate each logarithm. NO CALCULATOR!

a. \( \log_2 32 \) 
   \[ 2^5 = 32 \]

b. \( \log_3 1 \) 
   \[ 3^0 = 1 \]

c. \( \log_4 2 \) 
   \[ 4^{\frac{1}{2}} = 2 \]

d. \( \log_{100} \frac{1}{100} \) 
   \[ 10^{-2} = \frac{1}{100} \]

Example 5: Evaluate with the calculator. Round to 3 decimal places.

a. \( \log 25 \) 
   \[ 1.398 \]

b. \( \ln 0.34 \) 
   \[ -1.217 \]

c. \( \log x = 2.014 \) 
   \[ 10^{2.014} \approx 103.276 \]

d. \( \ln x = -4 \) 
   \[ 10^{-4} \approx 0.001 \]

e. \( \log x = 0 \) 
   \[ 10^0 = 1 \]

MEMORIZE: Change of Base Formula

The Change of Base Formula is used in order to evaluate a logarithm with a base other than 10 in the calculator. The Change-of-Base Formula is

\[ \log_b x = \frac{\log_a x}{\log_a b} \]

Example 6: Use the change of base formula to evaluate to 3 decimal places.

a. \( \log_2 15 \) 
   \[ \frac{\log 15}{\log 2} \approx 3.907 \]

b. \( \log_4 20 \) 
   \[ \frac{\log 20}{\log 4} \approx -2.161 \]

c. \( \log_{\frac{1}{5}} 1.5 \) 
   \[ \frac{\log 1.5}{\log \frac{1}{5}} \approx -4.531 \]