CONCEPT ONE: Graphing exponential functions.

Example 1: Graph the following functions on the grid provided. Then answer the questions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( y = 2^x )</td>
<td>A. What happens to the graph as the base increases? ( \boxed{\text{increases at a faster rate}} )</td>
</tr>
<tr>
<td>b. ( y = 3^x )</td>
<td></td>
</tr>
<tr>
<td>c. ( y = 4^x )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B. What is the y-intercept for each function? WHY? ( (0, 1) ) ( a^0 = 1 )</td>
</tr>
<tr>
<td></td>
<td>C. What are the x-intercepts? WHY? ( \boxed{\text{none}} ) ( b/c \ 0 \neq a^n ) ( \text{ever} )</td>
</tr>
<tr>
<td></td>
<td>D. What is the domain of each function? What is the range? ( D: \mathbb{R} \ \ R: (0, \infty) \ \ \text{or} \ \ y &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>E. Are the functions increasing or decreasing? One-to-one? ( \boxed{\text{increasing and one-to-one}} )</td>
</tr>
</tbody>
</table>

Example 2: Graph the following functions on the grid provided. Then answer the questions.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( y = 2^{-x} )</td>
<td>A. What happens to the graph as the base increases? ( \boxed{\text{decreases at a faster rate}} )</td>
</tr>
<tr>
<td>b. ( y = 3^{-x} )</td>
<td></td>
</tr>
<tr>
<td>c. ( y = \frac{1}{4^x} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B. What is the y-intercept for each function? WHY? ( (0, 1) )</td>
</tr>
<tr>
<td></td>
<td>C. What are the x-intercepts? WHY? ( \boxed{\text{none}} )</td>
</tr>
<tr>
<td></td>
<td>D. What is the domain of each function? What is the range? ( D: \mathbb{R} \ \ R: (0, \infty) \ \ \text{or} \ \ y &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>E. Are the functions increasing or decreasing? One-to-one? ( \boxed{\text{decreasing and one-to-one}} )</td>
</tr>
</tbody>
</table>

**GENERALIZATIONS FOR EXPONENTIAL FUNCTIONS:**

**Graph of \( y = a^x, \ a > 1 \):**

- **Domain:** \( (-\infty, \infty) \)
- **Range:** \( (0, \infty) \)
- **Intercept:** \( (0, 1) \)
- **Increasing:** \( x \)-axis is a horizontal asymptotes \( (a^x \rightarrow 0 \ \text{as} \ x \rightarrow \infty) \)
- **Continuous**

**Graph of \( y = a^{-x}, \ a > 1 \):**

- **Domain:** \( (-\infty, \infty) \)
- **Range:** \( (0, \infty) \)
- **Intercept:** \( (0, 1) \)
- **Decreasing:** \( x \)-axis is a horizontal asymptotes \( (a^{-x} \rightarrow 0 \ \text{as} \ x \rightarrow \infty) \)
- **Continuous**
CONCEPT TWO: Transformations of Exponential Functions.

Example 3: Graph the following functions on the grid provided.

- a. \( y = -2^x \)
- b. \( y = 2^{x+2} \)
- c. \( y = 2^x + 2 \)
- d. \( y = 0.5 \)
- e. \( y = 3^{x-1} \)
- f. \( y = 3^{x+4} - 5 \)

GENERALIZATIONS FOR TRANSFORMATIONS

<table>
<thead>
<tr>
<th>Horizontal Shift</th>
<th>Vertical Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = a^x ) to ( y = a^{x+c} )</td>
<td>( y = a^x ) to ( y = a^x + c )</td>
</tr>
<tr>
<td>Reflection in ( x )-axis</td>
<td>Reflection in ( y )-axis</td>
</tr>
<tr>
<td>( y = a^x ) to ( y = -a^x )</td>
<td>( y = a^x ) to ( y = -a^x )</td>
</tr>
</tbody>
</table>

CONCEPT THREE: Using the One-to-One Property to solve equations.

Example 4: Solve for \( x \).

- a. \( 9 = 3^{x+1} \)
  \[ 3^2 = 3^{x+1} \]
  \[ 2 = x + 1 \]
  \[ x = 1 \]

- b. \( 8^{x-3} = 4^{x+1} \)
  \[ \left(2^3\right)^{x-3} = \left(2^2\right)^{x+1} \]
  \[ 2^3x-9 = 2^2x+2 \]
  \[ 3x-9 = 2x+2 \]
  \[ x = 11 \]

- c. \( 5^{x+3} = \sqrt{125} \)
  \[ 5^{x+3} = 5^{\frac{3}{2}} \]
  \[ x + 3 = \frac{3}{2} \]
  \[ x = -\frac{3}{2} \]

- d. \( \left(\frac{1}{2}\right)^x = 8^{3x+6} \)
  \[ \left(2^{-1}\right)^x = (2^3)^{3x+6} \]
  \[ 2^{-x} = 2^{9x+18} \]
  \[ -x = 9x + 18 \]
  \[ 0 = x^2 + 9x + 18 \]
  \[ 0 = (x+6)(x+3) \]
  \[ x = -6, -3 \]
1. The population $P$ (in millions) of Russia from 1996 to 2004 can be approximated by the model $P = 152.26e^{-0.0039t}$, where $t$ represents the year, with $t = 6$ corresponding to 1996.

(a) According to the model, is the population of Russia increasing or decreasing? EXPLAIN.

In the form $y = b \cdot a^x$ with $a > 1$, therefore, it is decreasing.

(b) Find the population of Russia in 1998 and in 2000.

1998 $\rightarrow t = 8 \quad \approx 148$ million
2000 $\rightarrow t = 10 \quad \approx 140$ million

(c) Use the model to predict the population of Russia in 2010.

2010 $\rightarrow t = 20 \quad \approx 141$ million

2. Let $Q$ represent a mass of a radioactive radium (\(^{226}\text{Ra}\)) (in grams), whose half-life is 1599 years. The quantity of radium present after $t$ years is

$$Q = 25 \left( \frac{1}{2} \right)^{\frac{t}{1599}}$$

(a) Determine the initial quantity (when $t = 0$).

\[25\text{ grams}\]

(b) Determine the quantity present after 1000 years.

\[\approx 16.2\text{ grams}\]

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.

3. To estimate the amount of defoliation caused by the gypsy moth during a given year, a forester counts the number $x$ of egg masses on \(\frac{1}{40}\) of an acre (circle of radius 18.6 feet) in the fall. The percent of defoliation $y$ the next spring is shown in the table.

<table>
<thead>
<tr>
<th>Egg masses, $x$</th>
<th>Percent of defoliation, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>25</td>
<td>44</td>
</tr>
<tr>
<td>50</td>
<td>81</td>
</tr>
<tr>
<td>75</td>
<td>96</td>
</tr>
<tr>
<td>100</td>
<td>99</td>
</tr>
</tbody>
</table>

A model for the data is given by $y = \frac{100}{1 + 7e^{-0.0068x}}$.

(a) Use a graphing utility to create a scatter plot of the data and graph the model in the same window.

(b) Estimate the percent of defoliation if 36 egg masses are counted on \(\frac{1}{40}\) acre.

$x = 3.6$

\[\% \ 63.70\]

(c) You observe that $\frac{2}{3}$ of a forest is defoliated the following spring. Use the graph in part (a) to estimate the number of egg masses per \(\frac{1}{40}\) acre.

\[y = \frac{2}{3}(100) \quad \text{put in for } y\]

and calculate intersection

\[\% \ 38\text{ egg masses}\]