Section 9.5 - Binomial Theorem

DAY ONE: Several expansions of binomials may offer clues to a general formula for $(a+b)^n$.

$(a+b)^0 = 1$
$(a+b)^1 = 1a + 1b$
$(a+b)^2 = 1a^2 + 2ab + 1b^2$
$(a+b)^3 = 1a^3 + 3a^2b + 3a^1b^2 + 1b^3$
$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4a^1b^3 + 1b^4$

$(a+b)^5 = __________________________$

$(a+b)^6 = __________________________$

Let’s look for patterns that may be generalized to a formula for $(a+b)^n$.

Variables?

(1)______________________________________________________________________________

(2)______________________________________________________________________________

(3)______________________________________________________________________________

How to get the coefficients? USE PASCAL’S TRIANGLE!

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
```

Example 1) Use the formula for $(a+b)^4$ to find the expansion for $(2x+y)^4$. 
Example 2) Use the formula for \((a+b)^3\) to find the expansion for \((2x-5y)^3\).

Example 3) Use the formula for \((a+b)^6\) to find the first four terms in the expansion of \((x^2-y^3)^6\).

Now, do you remember the difference between permutations (P) and combinations (C)?

Ex. 4) \(7\ P_3 = \) ________________ or ________________

Do you remember how to do this using your calculator? ______________________________________

Ex. 5) How many ways can 10 contestants place 1st, 2nd, and 3rd in a contest?

__________________________________ or __________________________________________

Therefore… \(n\ P_k = \frac{n!}{(n-k)!}\)

Ex. 6) \(7\ C_3 = \) ________________ or ________________

Do you remember how to do this using your calculator? ______________________________________

Ex. 7) How many ways can a committee of 3 people be chosen from 10 people?

__________________________________ or __________________________________________

Therefore… \(n\ C_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}\)
Examples:

a. Find $\binom{15}{8}$.

b. Find $\binom{20}{6}$.

REMEMBER... $\binom{n}{0} = 1$ AND $\binom{n}{n} = 1$. (From Algebra II)

REMEMBER THESE?

\[
\begin{align*}
(a+b)^2 &= 1a^2 + 2ab + 1b^2 \\
(a+b)^3 &= 1a^3 + 3a^2b + 3ab^2 + 1b^3 \\
(a+b)^4 &= 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4 \\
(a+b)^5 &= \quad \\
(a+b)^{10} &= \quad \\
\end{align*}
\]

YUCK! Let's see if we can figure out a way to do the last one WITHOUT doing the expansions before it.

Put these values in your calculator in $y =$ and check the table values starting at $x = 0$:

\[
\begin{align*}
y &= 2 \binom{x}{2} & \quad \text{are the first 3 values.} \\
y &= 3 \binom{x}{3} & \quad \text{are the first 4 values.} \\
y &= 4 \binom{x}{4} & \quad \text{are the first 5 values.} \\
y &= 5 \binom{x}{5} & \quad \text{are the first 6 values.} \\
y &= 10 \binom{x}{10} & \quad \text{are the first 11 values.} \\
\end{align*}
\]

What do you notice? 

Could you use this to write the first 6 terms of $(a+b)^{10}$ above?

Binomial Theorem:

\[
(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n
\]
Examples:

a. Find \((x + y)^9\) using the Binomial Theorem.

b. Find the expansion for \((2x - 3y)^4\).

Could we generate the coefficients using our calculator, \(\text{y = }\), and table?

\[ Y = \left( \binom{4}{C_0} \right) \left( \frac{2^1}{1} \right) \left( \frac{-3}{1} \right) \]

C. Find the first 4 terms of \((x - 2y)^8\).

**DAY TWO:**

For the following expansion, what do you notice about relationship between the number of a term and the exponents and coefficient.

\[(a+b)^4 = 1^\text{st} + 2^\text{nd} + 3^\text{rd} + 4^\text{th} + 5^\text{th}\]

\[1a^4 + 4a^3b^1 + 6a^2b^2 + 4ab^3 + b^4\]

Thus - for \((a+b)^n\) ...

Last exponent in each term is always____________________________.

Exponents of the two terms always add to____________________________.

Coefficient is always __________________________________________.

because the coefficients are ____________________________.
Examples:

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<tr>
<td>a. Find the 9(^{\text{th}}) term of ((a + b)^{11}).</td>
<td>b. Find the 4(^{\text{th}}) term of ((2c + 3y^2)^{3}).</td>
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<tr>
<td>c. Find the 6(^{\text{th}}) term of ((a + 2b)^{8}).</td>
<td>d. Complete the term for (b^3) in ((a - b)^{10}).</td>
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<td>e. Find the coefficient of the term (a^6b^5) in the expansion of ((3a - 2b)^{11}).</td>
<td>f. The probability of a baseball player getting a hit during any given time at bat is (\frac{1}{4}). To find the probability that the player gets three hits during the next 10 times at bat, evaluate the term (\binom{10}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^7) in the expansion of (\left(\frac{1}{4} + \frac{3}{4}\right)^{10}).</td>
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