Section 9.3 – Geometric Sequences and Series

GEOMETRIC SEQUENCE: _________________________________________________________
________________________________________________________________________________
Common Ratio:  _______________________________________________________________

Today we are going to look for a "simple" way to find an explicit formula for a geometric sequence. Consider the following sequence. Do you see a pattern for obtaining each term?

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>15</td>
<td>45</td>
<td>135</td>
<td>405</td>
</tr>
<tr>
<td>5</td>
<td>(5X3)</td>
<td>(5X3)X3</td>
<td>((5X3)X3)X3</td>
<td>(((5X3)X3)X3)X3</td>
</tr>
<tr>
<td>$5(__)^0$</td>
<td>$5(__)^1$</td>
<td>$5(__)^2$</td>
<td>$5(__)^3$</td>
<td>$5(__)^4$</td>
</tr>
</tbody>
</table>

Therefore the **EXPLICIT FORMULA** is: $a_n = \______________________________$

What is the **RECURSIVE FORMULA**?

**General Form of a Geometric Sequence:**

Recursive: $a_1, a_{n+1} = a_n (r)$

Explicit: $a_n = a_1 \cdot r^{n-1}$

Example 1: For the following geometric sequences, find both the explicit and recursive formulas.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Recursive Formula AND Explicit Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 2, -12, 72, -432, ...</td>
<td>$a_1 = __________ \quad a_{n+1} = __________</td>
</tr>
<tr>
<td></td>
<td>$a_n = __________$</td>
</tr>
<tr>
<td>b) 9, 7.2, 5.76, 4.608, ...</td>
<td>$__________$</td>
</tr>
</tbody>
</table>
Example 2: Find the explicit formula and the indicated term.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>(a_4 = 64, \quad r = \frac{1}{4})</td>
</tr>
<tr>
<td></td>
<td>(a = \frac{1}{4}) (a_8 = \frac{1}{16})</td>
</tr>
<tr>
<td>b)</td>
<td>(5, 30, 180...)</td>
</tr>
<tr>
<td></td>
<td>(a_n = ) (a_{10} = \frac{1}{16})</td>
</tr>
<tr>
<td>c)</td>
<td>(a_2 = 3, \quad a_5 = \frac{3}{64})</td>
</tr>
<tr>
<td></td>
<td>(a_n = \frac{3}{64}) (a_1 = ) (\frac{1}{16})</td>
</tr>
<tr>
<td>d)</td>
<td>(a_3 = \frac{16}{3}, \quad a_5 = \frac{64}{27})</td>
</tr>
<tr>
<td></td>
<td>(a_n = \frac{16}{3}) (a_7 = ) (\frac{1}{16})</td>
</tr>
</tbody>
</table>

Could you find the 7th term without finding the general term formula?
Example 1: Find the indicated form and find the partial sum for the series.

\[
\sum_{n=1}^{5} 5 \cdot 2^{n-1}
\]

\[
\sum_{k=\text{___}}^{\text{____}} \text{____} + \text{____} + \text{____} + \text{____} + \text{____} \]

\[
S_7 = \text{__________}
\]

Just finding the terms and adding them up is good for series with a small number of terms. This is not a good method, however, if we have a large number of terms. And your teacher may ask you to find the sum of 100 terms... what a meanie!

We need a formula!

GEOMETRIC SERIES:

\[
S_n = \frac{a_1(1-r^n)}{1-r} \quad \text{for } r \neq 1
\]

Example 2: Find each indicated partial sum using the formula.

\[
\sum_{a=1}^{\infty} \frac{3}{4^a}
\]

\[
\sum_{k=\text{____}}^{\text{____}} \text{____} + \text{____} + \text{____} + \text{____} + \text{____} \]

\[
S_6 = \text{__________}
\]

Now let's look at finding the sum of a geometric series with an infinite number of terms.

Ex. 3) Find \( S_{\infty} \) for \( \sum_{k=1}^{\infty} 3(2)^{k-1} \).

\( S_{\infty} = \text{__________} \)

(Put the term formula in \( Y_1 \) and the sum formula in \( Y_2 \). Go to the table and see what happens to the terms, and the sum of the terms, as \( k \) gets larger.)

Can we find this sum? Why?
Ex. 4) Find $S_\infty$ for $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1}$. $S_\infty =$ ______________

(Put the term formula in $\sum_1^n$ and the sum formula in $\sum_2^n$. Go to the table and see what happens to the terms, and the sum of the terms, as $k$ gets larger.)

Can we find this sum? Why?

________________________________________________________________________________

________________________________________________________________________________

Why are we able to find the sum with one series and not the other?

________________________________________________________________________________

________________________________________________________________________________

Now, lets look at the general formula for finding the sum of a series and ask -

What would the formula become if $r$ is a number between -1 and 1 and $n$ is getting very large - approaching infinity?

$$S_n = \frac{a_1 \left(1 - r^n\right)}{1 - r} \quad = \quad \text{______________}$$

Therefore, the Sum of an Infinite Geometric Series for $|r| < 1$ is... $S = \frac{a_1}{1 - r}$.

Find the sums of the following infinite geometric series.

Ex. 5) $\sum_{n=1}^{\infty} 4(.06)^{n-1}$ $S =$ ______________

Ex. 6) $3 + 0.3 + 0.03 + 0.003 + \ldots$ $S =$ ______________