Pre-Calculus Notes   Name: ______________________

Section 2.2 – Polynomial Functions of Higher Degree

DAY ONE:   What is true about polynomial functions?

The Leading Coefficient Test/Beginning and Ending Behavior:
Positive lead with odd exponent = \( \Downarrow \Uparrow \)  \quad Negative lead with odd exponent = \( \Uparrow \Downarrow \)
Positive lead and even exponent = \( \Uparrow \Uparrow \)  \quad Negative lead with even exponent = \( \Downarrow \Downarrow \)

More Vocabulary:

A **turning point** is a point on a graph such that the value of the function is a relative maximum or relative minimum. This is when the graph switches from increasing to decreasing or from decreasing to increasing. **Relative extrema** consist of relative maximum and relative minimum values of the function. The graph of a polynomial of degree \( n \) can have as many as \( n - 1 \) relative extrema.

If \( f \) is a polynomial function and \( a \) is a real number, the following statements are equivalent.
1. \( x = a \) is a **zero** of the function \( f \).
2. \( x = a \) is a **solution** of the equation \( f(x) = 0 \).
3. \( (x-a) \) is a **factor** of \( f(x) \).
4. \( (a, 0) \) is an **x-intercept** of the graph of \( f \).

The **multiplicity** of a root \( r \) is the number of times that \( x - r \) is a **factor** of \( P(x) \).

When a real root has an **even** multiplicity, the graph of \( y = P(x) \) touches the **x-axis** but does not cross it. When a real root has **odd** multiplicity greater than 1, the graph “**bounces**” as it crosses the **x-axis**. In this class, we will primarily deal with multiplicities of 1, 2, or 3.

**End Behavior:** \( \Uparrow \Uparrow \)
Degree: even
Leading Coefficient: positive
Number of relative extrema: 3
Zeros and their Multiplicity:
- \( -3 \) w/ multiplicity of 2 (double root)
- \( 0 \) w/ multiplicity of 3 (triple root)
- \( 2 \) w/ multiplicity of 1 (single root)
Possible Function (factored form):
\[
y = +x^3 (x + 3)^2 (x - 2)
\]
Example 1: Find the requested information for each graph.

a. 

End Behavior: __________
Even or Odd Degree? __________
+ or - Leading Coeff.: __________
# of relative extrema: __________
Zeros and their Multiplicity:

Possible Function (factored form):

b. 

End Behavior: __________
Even or Odd Degree? __________
+ or - Leading Coeff.: __________
# of relative extrema: __________
Zeros and their Multiplicity:

Possible Function (factored form):

c. 

End Behavior: __________
Even or Odd Degree? __________
+ or - Leading Coeff.: __________
# of relative extrema: __________
Zeros and their Multiplicity:

Possible Function (factored form):

Example 2: Determine the MAXIMUM possible number of relative extrema for each polynomial function. State the beginning and ending behavior of the curve (⇑⇑, ⇑⇓, ⇓⇓, or ⇓⇑).

a.  \( f(x) = 3x^3 - 3x^2 + 2x - 1 \)

b.  \( f(x) = -x^3 + 2x-1 \)

Example 3: Factor each polynomial. Then identify its roots and their multiplicity along with the end behavior in order to sketch a graph WITHOUT the calculator.

a.  \( f(x) = x^4 - 29x^2 + 100 \)

b.  \( f(x) = -2x^5 - 5x^4 + 3x^3 \)
DAY TWO:

Example 4: Use a graphing utility to graph a sketch of the function. Determine each real zero AND the number of relative extrema. Then express the function as a product of linear factors.

a. \( f(x) = x^4 + 3x^3 - 4x \)

Relative Extrema: ___________

Real Zeroes (and their multiplicity):

Factored Form of Function:

b. \( f(x) = -2x^4 - 3x^3 + 9x^2 - x - 3 \)

Relative Extrema: ___________

Real Zeroes (and their multiplicity):

Factored Form of Function:

Example 5: Sketch a graph of the function WITHOUT using a calculator.

a. \( y = x(x+3)^3(x-2)^2 \)

b. \( y = -(x+4)^2(2x-1)(x-4) \)

Example 5: Determine a possible equation for the polynomial pictured in the graph.

Example 6: Miscellaneous Problems.

a. Find all the zeroes for \( f(x) = -\frac{1}{4}(x-3)^2(x+2)^3 \).

b. Sketch the graph of \( f(x) = 3x^4 - 4x^3 \).

c. Write an equation with degree \( n = 5 \) and having 2 as a triple root and 1 as a double root.