Part 1 - Misc. Function Concepts

Before we move on, we need to review what it means to find the composition of two functions.

**Composition of Two Functions** – \( f(g(x)) = (f \circ g)(x) \) where the domain is the set of all \( x \) in the domain of \( g \) such that \( g(x) \) is in the domain of \( f \).

Example 1: Given \( f(x) = x + 2 \) and \( g(x) = 4 - x^2 \), find...

<table>
<thead>
<tr>
<th>a. ((f \circ g)(-2))</th>
<th>b. (g(f(5)))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. (f(g(x)))</th>
<th>d. ((g \circ f)(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 2: Given the graphs, evaluate each of the following compositions.

<table>
<thead>
<tr>
<th>a. ((f \circ g)(1))</th>
<th>b. (g(f(-2)))</th>
<th>c. ((f \circ g)(-1))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Remember inverses?** Well, a function, \( f(x) \), has an inverse function, \( f^{-1}(x) \), if and ONLY if:

(a) The domain and range of \( f(x) \) are the same as the range and domain, respectfully, of \( f^{-1}(x) \).

(b) \( f(x) \) is a one-to-one function, which means it passes both the vertical AND horizontal line tests, or in a data chart for the function, the \( x \)-values do not repeat AND the \( y \)-values do not repeat.

(c) \( f(x) \) and \( f^{-1}(x) \) are reflections of each other in the line \( y = x \).

(d) \( f(f^{-1}(x)) = f^{-1}(f(x)) = x \)
Example 3: More with Inverses.

a. Verify by composition whether or not
   \[ f(x) = 3x - 6 \quad \text{and} \quad g(x) = \frac{x + 6}{3} \]
   are inverses of each other.

b. If \( f(x) \) is represented by the graph, give the domain and range (in interval notation) of its inverse. Also, is its inverse a function? Why or why not?

We can identify the intervals (\( x \)-values) over which a function is increasing, decreasing, or constant.

A function \( f \) is...
- **increasing** on an interval if, for any \( x_1 \) and \( x_2 \) in the interval, \( x_1 < x_2 \) implies \( f(x_1) < f(x_2) \).
- **decreasing** on an interval if, for any \( x_1 \) and \( x_2 \) in the interval, \( x_1 < x_2 \) implies \( f(x_1) > f(x_2) \).
- **constant** on an interval if, for any \( x_1 \) and \( x_2 \) in the interval, \( f(x_1) = f(x_2) \).

It is important to note that these intervals are **OPEN**. This means we only use parentheses, NO BRACKETS. This is because at the \( x \)-value where the function changes from increasing to decreasing, or from increasing to constant, or so on, the function is in the process of changing direction.

Example 4: Identify the intervals where the function is increasing, decreasing, and constant.

Part 2 - Transformations of Parent Graphs

**Shift** \((h, k)\) ⇒ Left or Right \(h\)  \quad Horizontal Shift  \quad Down or Up \(k\)  \quad Vertical Shift

Recall from Algebra II: Given \( f(x) = x^2 \), identify the transformations for the following...

a. \( g(x) = (x - 3)^2 \)  
b. \( h(x) = x^2 - 3 \)  
c. \( j(x) = (x + 4)^2 + 5 \)

So what about functions that are NOT quadratic?

\[
\begin{align*}
    f(x) &= (x-h)^2 + k \\
    f(x) &= |x-h| + k \\
    f(x) &= (x-h)^3 + k \\
    f(x) &= \sqrt{x-h} + k \\
    f(x) &= \frac{1}{x-h} + k
\end{align*}
\]
Example 1: Write the equation of each graph.

![Graphs A, B, C]

Reflecting Graphs ⇒ reflection in the $x$-axis $h(x) = -f(x)$
reflection in the $y$-axis $h(x) = f(-x)$

Example 2: Graph each function.

![Graphs a, b, c]

Vertical Shrink or Stretch ⇒ $y = c \cdot f(x)$
x-values stay the same, y-values are multiplied by $c$

Example 3: Graph each function.

![Graphs a, b, c]
**Horizontal Shrink or Stretch**  
\[ y = f \left( c \cdot x \right) \]
- \(x\)-values divided by \(c\), \(y\)-values stay the same

**Example 4:** Graph each function.

a. \( f(3x) \) given \( f(x) = \sqrt{x} \)

b. \( f\left(\frac{1}{3}x\right) \) given \( f(x) = \sqrt{x} \)

---

**Given the graph of \( f(x) \), graph the following transformations. LABEL or use different COLORS.**

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Graph 1</th>
<th>Graph 2</th>
<th>Graph 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( y = -f(x) )</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>(b) ( y = f(-x) )</td>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>(c) ( y = f(x) - 2 )</td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
<td><img src="image9" alt="Graph" /></td>
</tr>
<tr>
<td>(d) ( y = f(x + 1) )</td>
<td><img src="image10" alt="Graph" /></td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
</tr>
<tr>
<td>(e) ( y = 2f(x) )</td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Graph" /></td>
<td><img src="image15" alt="Graph" /></td>
</tr>
<tr>
<td>(f) ( y = f(2x) )</td>
<td><img src="image16" alt="Graph" /></td>
<td><img src="image17" alt="Graph" /></td>
<td><img src="image18" alt="Graph" /></td>
</tr>
<tr>
<td>(g) ( y = \frac{1}{2}f(x) )</td>
<td><img src="image19" alt="Graph" /></td>
<td><img src="image20" alt="Graph" /></td>
<td><img src="image21" alt="Graph" /></td>
</tr>
<tr>
<td>(h) ( y = f\left(\frac{1}{2}x\right) )</td>
<td><img src="image22" alt="Graph" /></td>
<td><img src="image23" alt="Graph" /></td>
<td><img src="image24" alt="Graph" /></td>
</tr>
</tbody>
</table>