Even and Odd Functions, Piece-Wise Functions, and the Greatest Integer Function

Even functions are symmetric with the y-axis. (Fold across the y-axis.)
Odd functions are symmetric with the origin. (Rotate 180 degrees about the origin.)

Example 1: Determine whether each function is even, odd, or neither based on symmetry. EXPLAIN.

For Even functions: \( f(-x) = f(x) \) (Plug in \(-x\) into the function and you get the SAME.)
For Odd functions: \( f(-x) = -f(x) \) (Plug in \(-x\) into the function and you get the OPPOSITE.)

Example 2: Determine ALGEBRAICALLY whether each function is even, odd, or neither.

a. \( g(x) = x^3 - x \)
   b. \( h(x) = x^2 + 1 \)
   c. \( f(x) = x^3 + x - 3 \)

Example 3: Given the following information, evaluate each of the following...

a. If \( f(x) \) is odd and \( f(2) = -3 \), then find \( f(-2) \).
   b. If \( f(x) \) is even and \( f(-6) = -0.5 \), then find \( f(6) \).
   c. If \( f(x) \) is odd and \( f(-1) = -5 \), then find \( f(1) \).
Piecewise Function – A function made up of two or more equations with given domains for each. The "pieces" are graphed one at a time.

\[ f(x) = \begin{cases} 
  x + 1, & x \geq 0 \\
  -x^2, & x < 0 
\end{cases} \]

**Example 1:** Evaluate the function \( f(x) = \begin{cases} 
  x + 1, & x \geq 0 \\
  -x^2, & x < 0 
\end{cases} \) for the following \( x \)-values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
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<tbody>
<tr>
<td>a. ( x = -1 )</td>
<td>( f(x) = -1^2 = -1 )</td>
</tr>
<tr>
<td>b. ( x = 0 )</td>
<td>( f(x) = 0 + 1 = 1 )</td>
</tr>
<tr>
<td>c. ( x = 1 )</td>
<td>( f(x) = 1 + 1 = 2 )</td>
</tr>
<tr>
<td>d. ( x = 4 )</td>
<td>( f(x) = 4 + 1 = 5 )</td>
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**Example 2:** Graph each piece-wise function.

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<tr>
<th>( x )</th>
<th>( f(x) )</th>
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| a. \( f(x) = \begin{cases} 
  2x + 3, & x \leq 1 \\
  -x + 4, & x > 1 
\end{cases} \) |
| b. \( f(x) = \begin{cases} 
  -(x + 2)^2, & x \leq -1 \\
  |x| + 4, & x > -1 
\end{cases} \) |
Greatest Integer Function - \( f(x) = [x] \) Reads as “\( f(x) \) is equivalent to the greatest integer less than or equal to \( x \).”

\([-2.75] = “The greatest integer less than or equal to -2.75.”

Example 3: Evaluate each of the following for \( f(x) = [x] \).

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<tbody>
<tr>
<td>a. ( f(5) )</td>
<td>b. ( f(-3.2) )</td>
<td>c. ( f(1.9) )</td>
<td>d. ( f(-0.5) )</td>
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So what does the graph of \( f(x) = [x] \) look like?

The domain of the function is _____________. The range of the function is _____________. The graph has a \( y \)-intercept at _____________ and \( x \)-intercepts on the interval _____________. The graph is _____________ between each pair of consecutive integers. The graph _____________ vertically ________ unit(s) at each integer value. The closed dot is on the _____________ and the open dot is on the _____________.

Example 4: Graph each function. Use what you learned about transformations of parent graphs.

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<tbody>
<tr>
<td>a. ( f(x) = [x + 2] - 1 )</td>
<td>b. ( f(x) = -2[x] )</td>
<td>c. ( f(x) = \frac{1}{3} x + 2 )</td>
</tr>
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