Pre-Calculus Notes

Section 6.1 - Intro to the Law of Sines
Section 6.2 - Areas of Triangles

The LAW of SINES... what is its use? solving for sides and angles of △

When given 2 angles and 1 side, or 2 sides and non-included angle.

AAS
ASA
SSA

MEMORIZE: THE LAW OF SINES

* to be used given AAS, ASA, or SSA

For ANY triangle \( \triangle ABC \), where \( a \), \( b \), and \( c \) are the lengths of the sides OPPOSITE the angles with measures \( A \), \( B \), and \( C \) (respectively),

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.
\]

Law of Sines

Example 1: Solve \( \triangle ABC \).

a. Note: We are given ASA here.

\[
sin \ 81^\circ = \frac{sin 42^\circ}{b}
\]

\[
a \sin 81^\circ = 67 \sin 42^\circ
\]

\[
a = \frac{67 \sin 42^\circ}{\sin 81^\circ}
\]

\[
a \approx 45
\]

\[
c = 180 - 42 - 57
\]

\[
C = 81^\circ
\]

b. Note: We are given AAS here.

\[
sin 23^\circ = \frac{sin 87^\circ}{a}
\]

\[
a \sin 87^\circ = 47.5 \sin 23^\circ
\]

\[
a = \frac{47.5 \sin 23^\circ}{\sin 87^\circ}
\]

\[
a \approx 18.6
\]

\[
\sin 70^\circ = \frac{sin 87^\circ}{a}
\]

\[
b \sin 87^\circ = 47.5 \sin 70^\circ
\]

\[
b = \frac{47.5 \ sin 70^\circ}{\sin 87^\circ}
\]

\[
b \approx 44.7
\]
Example 2: WORD PROBLEM.

A ship is moving in a straight line towards the Point Cove lighthouse. The measure of the angle of elevation from the bridge of the ship to the lighthouse beacon is 25°. Later, from a point 600 feet closer, the angle of elevation is 47°. To the nearest foot, how high is the beacon above the level of the bridge of the ship?

\[
\sin 25° = \frac{\sin 22°}{y} = \frac{600 \sin 25°}{\sin 22°}
\]

Store in calculator!

\[
\sin 47° = \frac{x}{y}
\]

\[
x = y \sin 47°
\]

\[
x \approx 495 \text{ ft}
\]

Example 3: WORD PROBLEM.

The bearing from the Pine Knob fire tower to the Colt Station fire tower is N 65° E, and the two towers are 30 kilometers apart. A fire spotted by rangers in each tower has a bearing of N 80° E from Pine Knob and S 65° E from Colt Station. Find the distance of the fire from the Pine Knob tower.

\[
\sin 35° = \frac{\sin 130°}{x}
\]

\[
x = \frac{30 \sin 130°}{\sin 35°}
\]

\[
x \approx 40 \text{ km}
\]

Do you remember the formula for finding the area of a triangle given SAS? 

\[A = \frac{1}{2}bh\]

\[K = \frac{1}{2}b \cdot c \sin A\]

\[\sin A = \frac{h}{c}\]

\[h = c \sin A\]

The area, \(K\) of triangle \(ABC\) is given by any one of these formulas:

\[K = \frac{1}{2}bc \sin A\]

\[K = \frac{1}{2}ac \sin B\]

\[K = \frac{1}{2}ab \sin C\]
We also have a formula for finding the area of a triangle given SSS. (The Greek mathematician Heron developed the formula—hence it is called HERON'S AREA FORMULA.)

The area, \( K \) of triangle \( ABC \) is given by:

\[
K = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}.
\]
s is called the semi-perimeter of the triangle.

Example 1: Determine the area of \( \triangle DEF \) to the nearest square inch. DRAW A PICTURE.

\( d = 15.2, \ e = 22.7, \text{ and } f = 8.9 \)

\[
S = \frac{15.2 + 22.7 + 8.9}{2} = 23.4
\]

\( K \approx 44 \text{ in}^2 \)

Store! (Saves time)
(as \( x \))

Example 2: Which formula would you use to find the area of the following triangles? Find the area.

a. SAS

\[
K = \frac{1}{2} \cdot b \cdot h
\]

\( K \approx 4.6 \text{ u}^2 \)

b. \( S = \frac{s + t + \frac{s}{2}}{2} = 6.5 \)

\[
K = \sqrt{s(s-a)(s-b)(s-c)}
\]

\( K \approx 5.3 \text{ u}^2 \)

c. \( \triangle \)

\( K = \frac{1}{2} \cdot b \cdot h \)

\( K \approx 12 \text{ u}^2 \)

d. \( \triangle \) Law of Sines to find side length

\[
\sin 50^\circ = \frac{x}{10} \quad x = \frac{10 \sin 50^\circ}{\sin 100^\circ}
\]

\( K \approx 19.4 \text{ u}^2 \)

Example 3:

You want to buy a triangular lot measuring 1350 feet by 1860 feet by 2490 feet. The price of the land is $2200 per acre. How much does the land cost? (1 acre = 43,560 square feet)

\[
S = \frac{P}{2} = \frac{1350 + 1860 + 2490}{2} = 2850
\]

\[
K = \sqrt{2850 \cdot 1500 \cdot 990 \cdot 360}
\]

\( K \approx 1234345.981 \text{ ft}^2 \)

\( \frac{43560}{\text{ft}^2} \)

\( K \approx 28.33668489 \text{ acres} \times 2200 \)

Cost \( \approx \$ 62,340.71 \)