Section 4.1 - Radian and Degree Measure

PART ONE: Radians and Degrees

Angle in Standard Position:

Types of Angles:

There are 360 degrees in ONE revolution.

Example 1: Sketch the following angles in standard position.

a. 245°

b. -170°

Radian Measure: One radian is the measure of a central angle that intercepts an arc equal to the radius of the circle.

Draw in an angle with measure of "about" one radian.

Approximately how many radians are in a circle? \( \approx \) \( \text{______ radians} \)

EXACTLY how many radians are in a circle? \( = \frac{2\pi}{\text{______ radians}} \)

What is circumference of a circle? \( C = \frac{2\pi r}{\text{______}} \)

This means that one revolution is equal to \( 2\pi \) radians.
Example 2: Sketch the following angles in standard position.

<table>
<thead>
<tr>
<th></th>
<th>15π/4 = 3 3π/4</th>
<th>-2π/3</th>
<th>4 rads</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
</tr>
</tbody>
</table>

Example 3: State the quadrant or axis where each angle terminates.

<table>
<thead>
<tr>
<th></th>
<th>157°</th>
<th>-75°</th>
<th>252°</th>
<th>-390°</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
<tr>
<td>b.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 252°</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
<tr>
<td>d. -390°</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
<tr>
<td>e. -2π</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
<tr>
<td>f. 5π/3</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
<tr>
<td>g. 3.75</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
<tr>
<td>h. -5.12</td>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
<td><img src="image-d" alt="Image" /></td>
</tr>
</tbody>
</table>

**Co-terminal Angles** are angles that have the *same initial and terminal sides*.

Draw in three other angles co-terminal with the given angle.

<table>
<thead>
<tr>
<th></th>
<th>120° - 360°</th>
<th>120° + 360°</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
</tr>
</tbody>
</table>

How many angles are co-terminal with 120°? *infinite*

Could you get the calculator to give you a list of ALL of them? *Sort of: y = 120 + 360x, then look @ table*

<table>
<thead>
<tr>
<th></th>
<th>5π/4 - 2π</th>
<th>5π/4 + 2π</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image-a" alt="Image" /></td>
<td><img src="image-b" alt="Image" /></td>
<td><img src="image-c" alt="Image" /></td>
</tr>
</tbody>
</table>

How many angles are co-terminal with 5π/4 radians? *infinite*

Could you get the calculator to give you a list of ALL of them? *Sort of* In π form? *__________*

Yes, if __________.
**Example 4:** State two coterminal angles for each given angle… one positive and one negative.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (40^\circ)</td>
<td>b. (-800^\circ)</td>
</tr>
</tbody>
</table>
| \(40 + 360 = 400^\circ\) | \(-800 + 360 = -440^\circ\) | \(
\frac{5\pi}{4} - 2\pi = \frac{-3\pi}{4}
\) |
| \(40 - 360 = -320^\circ\) | \(-800 + 5(360) = 280^\circ\) | \(
\frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}
\) |

d. \(9\pi\) | e. \(5\) | f. \(-47^\circ\) |
| \(9\pi - 2\pi = 7\pi\) | \(5 - 2\pi \approx -1.28\) | \(-47 + 360 = 313^\circ\) |
| \(9\pi - 2(\pi) = -\pi\) | \(5 + 2\pi \approx 11.28\) | \(-47 - 360 = -407^\circ\) |

**PART TWO: Conversions Between Degrees and Radians**

1 revolution is \(2\pi\) radians

\[
\frac{2\pi \text{ rad}}{360^\circ} = \frac{\pi}{180^\circ}
\]

SO 1 radian = \(\frac{180^\circ}{\pi}\)

then 7 radians = \(7 \times \frac{180^\circ}{\pi}\)

To convert rads \(\rightarrow\) deg, multiply rads by \(\frac{180}{\pi}\) degrees.

**Example 1:** Convert from degrees to radians.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (135^\circ)</td>
<td>b. (540^\circ)</td>
</tr>
<tr>
<td>(\frac{135^\circ}{1} \times \frac{\pi}{180^\circ} = \frac{3\pi}{4} \text{ rad})</td>
<td>(\frac{540^\circ}{1} \times \frac{\pi}{180^\circ} = \frac{3\pi}{2} \text{ rad})</td>
</tr>
</tbody>
</table>

**Example 2:** Convert from radians to degrees.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (\frac{\pi}{2})</td>
<td>b. (\frac{9\pi}{2})</td>
</tr>
<tr>
<td>(\frac{-\pi \text{ rad}}{2} \div \frac{\pi}{\pi \text{ rad}} = -90^\circ)</td>
<td>(\frac{9\pi \text{ rad}}{2} \div \frac{180^\circ}{\pi \text{ rad}} = 810^\circ)</td>
</tr>
</tbody>
</table>

Can we divide an angle into units smaller than a degree? **Yes**

Why would we want to do so? **Better accuracy**
Deg° Min' Sec"  OR  D° M' S"

1 degree can be divided into 60 minutes (NOT TIME), so  

1' = \frac{1}{60} \text{ deg.}

1 minute can be divided into 60 seconds (NOT TIME), so  

1" = \frac{1}{60} \text{ min.}

So, there are 3600 seconds in 1 degree, resulting in  

1" = \frac{1}{3600} \text{ deg.}

Example 3: Change each angle measure to decimal degree form. SHOW YOUR WORK.

a. 42° 33' 40"

\[
\begin{align*}
42° + \frac{33'}{60} + \frac{40''}{3600} &= 42° + 0.55° + 0.0111° \\
\ &= 42.56°
\end{align*}
\]

b. -300° 52' 30"

\[
\begin{align*}
-300° + \frac{52'}{60} + \frac{30''}{3600} &= -300° - 0.87° - 0.0083° \\
\ &= -300.878°
\end{align*}
\]

Example 4: Change each angle measure to D° M' S" form. SHOW YOUR WORK.

a. 2.54°

\[
\begin{align*}
2° + 0.54° &= 2° + \frac{54'}{60} \\
2° + 0.9° &= 2° + 0.15° \\
2° + 32.4' &= 2° + 32' + 0.4' \\
2° + 32' + 0.4' &= 2° + 32' + \frac{4'}{60} \\
2° + 32' + 0.4' &= 2° + 32' + \frac{4'}{60} \\
\ &= 2° 32' 24"
\end{align*}
\]

b. -36.126°

\[
\begin{align*}
-36° + \frac{12.6'}{60} &= -36° + 0.21° \\
-36° + 0.21° &= -36° + 0.36' \\
-36° + 0.36' + \frac{6'}{60} &= -36° + 0.36' + \frac{6'}{60} \\
-36° + 0.36' + \frac{6'}{60} &= -36° + 0.36' + \frac{6'}{60} \\
\ &= -36° 7' 33.6"
\end{align*}
\]

Will the calculator do the computations above for us?  yes

How?  look under angle menu  →  2nd  APPS  

and " is  ALPHA +1