DAY ONE: What is true about polynomial functions?

(a) Polynomial functions have continuous graphs.
(b) Functions with graphs that are not continuous are not polynomial functions.

Polynomial functions have graphs with smooth rounded turns.

The Leading Coefficient Test/Beginning and Ending Behavior:
Positive lead with odd exponent = \( \downarrow \uparrow \)  
Negative lead with odd exponent = \( \uparrow \downarrow \)

Positive lead and even exponent = \( \uparrow \uparrow \)  
Negative lead with even exponent = \( \downarrow \downarrow \)

More Vocabulary:

A turning point is a point on a graph such that the value of the function is a relative maximum or relative minimum. This is when the graph switches from increasing to decreasing or from decreasing to increasing. Relative extrema consist of relative maximum and relative minimum values of the function. The graph of a polynomial of degree \( n \) can have as many as \( n-1 \) relative extrema.

If \( f \) is a polynomial function and \( a \) is a real number, the following statements are equivalent.
1. \( x = a \) is a zero of the function \( f \).
2. \( x = a \) is a solution of the equation \( f(x) = 0 \).
3. \( (x-a) \) is a factor of \( f(x) \).
4. \( (a, 0) \) is an \( x \)-intercept of the graph of \( f \).

The multiplicity of a root \( r \) is the number of times that \( x-r \) is a factor of \( P(x) \).

When a real root has an even multiplicity, the graph of \( y = P(x) \) touches the \( x \)-axis but does not cross it. When a real root has odd multiplicity greater than 1, the graph "bends" as it crosses the \( x \)-axis. In this class, we will primarily deal with multiplicities of 1, 2, or 3.

End Behavior: \( \uparrow \uparrow \)  
Degree: even
Leading Coefficient: positive
Number of relative extrema: 3
Zeros and their Multiplicity: 
-3 w/ multiplicity of 2 (double root)
0 w/ multiplicity of 3 (triple root)
2 w/ multiplicity of 1 (single root)
Possible Function (factored form):
\[ y = +x^3(x+3)^2(x-2) \]
Example 1: Find the requested information for each graph.

a. End Behavior: \[ \longrightarrow \]
   Even or Odd Degree? \[\text{even}\]
   + or - Leading Coeff.: \[\frac{1}{2}\]
   # of relative extrema: \[5\]
   Zeros and their Multiplicity:
   \(-3\) triple
   \(-1\) single
   \(\frac{1}{2}\) single
   4 double
   Possible Function (factored form):
   \[ y = -(x+3)^3(2x-5)(x+1)(x+4)^2 \]

b. End Behavior: \[ \uparrow \downarrow \]
   Even or Odd Degree? \[\text{odd}\]
   + or - Leading Coeff.: \[\frac{1}{2}\]
   # of relative extrema: \[4\]
   Zeros and their Multiplicity:
   \(-2\) single
   \(-1\) single
   0 double
   2 single
   Possible Function (factored form):
   \[ y = -x^2(x+2)(x+1)(x-2) \]

c. End Behavior: \[ \uparrow \downarrow \]
   Even or Odd Degree? \[\text{even}\]
   + or - Leading Coeff.: \[\frac{1}{2}\]
   # of relative extrema: \[3\]
   Zeros and their Multiplicity:
   \(-1\) single
   \(-\frac{1}{2}\) triple
   \(1\) double
   Possible Function (factored form):
   \[ y = +(x+1)(2x+1)^3(x-1) \]

Example 2: Determine the **maximum** number of relative extrema for each polynomial function. State the beginning and ending behavior of the curve (\(\uparrow\uparrow, \uparrow\downarrow, \downarrow\downarrow, \text{ or } \downarrow\uparrow)\).

a. \( f(x) = 3x^2 - 3x^2 + 2x - 1 \) \[\text{odd}\]
   \[5 - 1 = 4\] \[\downarrow \uparrow\]

b. \( f(x) = -x^2 + 2x - 1 \) \[\text{odd}\]
   \[3 - 1 = 2\] \[\uparrow \downarrow\]

Example 3: Factor each polynomial. Then identify its roots and their multiplicity along with the end behavior in order to sketch a graph WITHOUT the calculator.

a. \( f(x) = x^4 - 29x^2 + 100 \)
   \[ f(x) = (x^2 - 25)(x^2 - 4) \]
   \[ f(x) = (x - 5)(x + 5)(x - 2)(x + 2) \]
   \[\uparrow \uparrow\]

b. \( f(x) = 2x^2 - 5x^3 + 3x^3 \)
   \[ f(x) = -x^3(2x - 5)(x - 3) \]
   \[ f(x) = -x^3(2x - 1)(x + 3) \]
   \[\text{triple} \ \frac{1}{2} \ -3\]

\[\text{Graphs}\]
DAY TWO:

Example 4: Use a graphing utility to graph a sketch of the function. Determine each real zero AND the number of relative extrema. Then express the function as a product of linear factors.

a. \( f(x) = x^4 + 3x^3 - 4x \)
   
   \# Relative Extrema: 3
   
   Real Zeros (and their multiplicity):
   
   -2 double 0 single 1 single
   
   Factored Form of Function:
   
   \( f(x) = (x+2)^2 (x-1) \)

b. \( f(x) = 2x^4 - 3x^3 + 9x^2 - x - 3 \)
   
   \# Relative Extrema: 3
   
   Real Zeros (and their multiplicity):
   
   -3 single -\( \frac{1}{2} \) single 1 double
   
   Factored Form of Function:
   
   \( f(x) = -(x+3)(x+1) \)

Example 5: Sketch a graph of the function WITHOUT using a calculator.

a. \( y = \frac{1}{4} x(x+3)^3 (x-2)^2 \)
   
   \( \Omega \) -3 triple 2 double

b. \( y = (x+4)^2 (2x-1)(x-4) \)
   
   \( \Omega \) -4 double \( \frac{1}{2} \) 4

Example 5: Determine a possible equation for the polynomial pictured in the graph.

a. \( y = x^2 (x+2)^2 (2x+1)(x-1) \)

b. \( y = - (x+1)^3 (x-2)^2 \)

Example 6: Miscellaneous Problems. ©

a. Find all the zeroes for \( f(x) = -\frac{1}{4} (x-3)^2 (x+2)^3 \).
   
   3 (double)
   
   -2 (triple)

b. Sketch the graph of \( f(x) = 3x^4 - 4x^3 \).
   
   \( \Omega \) triple \( \frac{1}{3} \)

   
   End behavior not specified.

   c. Write an equation with degree \( n = 5 \) and having 2 as a triple root and 1 as a double root.
   
   \( y = + (x-2)^3 (x-1)^2 \) or
   
   \( y = - (x-2)^3 (x-1)^2 \)