Pre-Calculus Notes
Section 10.2 - Parabolas Day 2

Parametric Form of Parabola Equations:

\[ 4c(y-k) = (x-h)^2 \] becomes \( x = T \) and \( y = \frac{1}{4c}(T-h)^2 + k \) in parametric form.

\[ 4c(x-h) = (y-k)^2 \] becomes \( x = \frac{1}{4c}(T-k)^2 + h \) and \( y = T \) in parametric form.

Why would we even want to deal with parametric form of the parabola?

Graph \( 16(x+3) = (y-1)^2 \) on your graphing calculator.

\[ \sqrt{16(x+3)} = \sqrt{(y-1)^2} \]
\[ y - 1 = \pm \sqrt{16(x+3)} \]
\[ y = 1 \pm \sqrt{16(x+3)} \]

Example 1: Rewrite each parabola in standard form AND parametric form.

a. \( y = -\frac{1}{8}x^2 + 2 \)

\[ y - 2 = -\frac{1}{8}x^2 \]

\[ -8(y-2) = x^2 \]

\[ x = T \]
\[ y = \frac{1}{8}T^2 + 2 \]

b. \( y^2 - 2x + 2y + 7 = 0 \)

\[ \frac{y^2 + 2y + \frac{1}{4}}{2} = \frac{2x + 7 + \frac{1}{4}}{2} \]

\[ (y + 1)^2 = 2x + 8 \]

\[ (y+1)^2 = 2(x+4) \]

\[ x = \frac{1}{2}(T+1)^2 - 4 \]
\[ y = T \]
Example 2: Write the equation of the parabolas in both standard AND parametric form given the following information.

a. Vertex at the origin and directrix \( y = 1 \)

- Opens down
- \( C = -1 \)
- \( x^2 \)

\[
4c(y-k) = (x-h)^2 \\
-4(y-0) = (x-0)^2 \\
x = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \\
y = -\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}
\]

b. Vertex at \((5,2)\) and focus \((3,2)\)

- Opens left
- \( C = -2 \)
- \( y^2 \)

\[
4c(x-h) = (y-k)^2 \\
-8(x-5) = (y-2)^2 \\
x = \frac{-1}{8} (5-3)^2 + 5 \\
y = 5
\]

c. Focus at \((2,2)\) and directrix \( x = -2 \)

- Vertex \((0,2)\)
- Opens right
- \( C = 2 \)
- \( y^2 \)

\[
4c(x-h) = (y-k)^2 \\
8(x-0) = (y-2)^2 \\
x = \frac{1}{8} (2-0)^2 \\
y = 2
\]

d. \( x^2 \)

- Opens down
- Vertex \((3,1)\)

\[
4c(y-k) = (x-h)^2 \\
-4(y-1) = (x-3)^2 \\
x = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \\
y = -\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}
\]

e. \( y^2 \)

- Opens right
- Vertex \((2,-2)\)

\[
4c(x-h) = (y-k)^2 \\
8(x-2) = (y+2)^2 \\
x = \frac{1}{8} (2+2)^2 + 2 \\
y = 2
\]